# Long Hedge

Suppose:

$F_1$: Initial Futures Price

$F_2$: Final Futures Price

$S_2$: Final Asset Price

- You hedge the future purchase of an asset by entering into a long futures contract
- Cost of Asset=$S_2 - (F_2 - F_1) = F_1$ + Basis

# Long Hedge

- $F_1$: \$32.5 at time period t=1
- $F_2$: \$34.6 at time period t=2
- $S_2$: \$35.0 at time period t=2
- Now you hedge the future purchase of an asset by entering into a long futures contract, so the cost for the asset incurred you is {35.0 – (34.6 – 32.5)} = ***32.9*** = {32.5 + (35.0 – 34.6)} = {32.5 + 0.4}, which is the total cost incurred by you to operate in two markets, where one market is the spot while the other is the futures
- On the other hand if you has operated only in the spot market then the cost incurred by you for buying the asset would have been ***35.0***

# Short Hedge

Suppose that:

$F_1$: Initial Futures Price

$F_2$: Final Futures Price

$S_2$: Final Asset Price

- You hedge the future sale of an asset by entering into a short futures contract
- Price Realized=$S_2$+ ($F_1$ –$F_2$) = $F_1$ + Basis

# Short Hedge

- $F_1$: \$32.5 at time period t=1
- $F_2$: \$29.8 at time period t=2
- $S_2$: \$35.0 at time period t=2
- Now you hedge the future selling of an asset by entering into a short futures contract, so the price realized by you from the asset is {35.0 + (32.5 – 29.8)} = **37.7** = {32.5 + 5.2} = {32.5 + (35.0 – 29.8)}, which is the total price realized by you to operate in two markets, where one market is the spot while the other is the futures
- On the other hand if you has operated only in the spot market then the price realized by you from selling the asset would have been **35.0**

# Long or Short Hedge

- Suppose that

  $F_1$ : Initial Futures Price

  $F_2$ : Final Futures Price

  $S_2$ : Final Asset Price

  $S^*_2$: Final price of Asset

- Cost of Asset=$S_2 + F_1 - F_2 = F_1 + (S^*_2 - F_2) + (S_2 - S^*_2)$ or

  $$S_2 - (F_2 - F_1) = F_1 + (S^*_2 - F_2) + (S_2 - S^*_2)$$

  which are same.

# Long Hedge

- $F_1$: $42.8
- $F_2$: $43.4
- $S_2$: $44.0
- $S^*_2$: $42.7
- Cost of the asset (considering we are in the long position) = {44.0 + 42.8 - 43.4} = {42.8 +(42.7 – 43.4) + (44.0 – 42.7) = *43.4*
- On the other hand if you has operated only in the spot market then the price realized by you from selling the asset would have been *42.7*

# Optimal Hedge Ratio

***<u>It is the ratio of the size of the position taken in futures contracts to the size of the exposure in the actual asset market</u>***

If $\sigma_S$ and $\sigma_F$ are SD of $\delta S$ and $\delta F$ and $\rho$ is coefficient of correlation between $\delta S$ (change in spot price, S, during a time which is equal to the life of the hedge) and $\delta F$ (change in futures price, F, during the time which is equal to the life of the hedge).

Then

- For Short Futures the change in value of hedge's position during the life of the hedge is: $\delta S - h\delta F$
- For Long Futures the change in value of hedge's position during the life of the hedge is : $h\delta F - \delta S$

# Optimal Hedge Ratio

Proportion of the exposure that should optimally be hedged is

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

where

- $\sigma_S$ is the standard deviation of $\delta S$, the change in the spot price during the hedging period,
- $\sigma_F$ is the standard deviation of $\delta F$, the change in the futures price during the hedging period
- $\rho$ is the coefficient of correlation between $\delta S$ and $\delta F$.

# Hedging Using Index Futures

- To hedge the risk in a portfolio the number of contracts that should be shorted is

$$\beta \frac{P}{A}$$

- where $P$ is the value of the portfolio, $\beta$ is its beta, and $A$ is the value of the assets underlying one futures contract

# Rolling the Hedge Forward

***Rollover basis***

- Time $t_1$: Short futures contract 1
- Time $t_2$: Close out future contract 1
  Short futures contract 2

.

.

- Time $t_n$: Close out futures contract (n-1)
  Short futures contract n
- Time T: Close out futures contract n

# Comparison between Forward and Future contracts

| Forwards | Futures |
|---|---|
| 1. Private contracts between two parties | 1. Traded on an exchange |
| 2. Not standardized | 2. Standardized contract |
| 3. Usually one specified delivery | 3. Range of delivery dates |
| 4. Settled at the end of the Contract | 4. Settled daily |
| 5. Delivery or final cash Settlement usually takes place | 5. Contract usually closed out prior to maturity |

# Example 2.1

Suppose that you enter into a short futures contract to sell July silver for $5.20 per ounce on the New York Commodity Exchange. The size of the contract is 5000 ounces. The initial margin is $4000 and the maintenance margin is $3000. What changes in the futures price will lead to a margin call? What happens if you do not meet the margin call?

# Example 2.2

Suppose the standard deviation of quarterly changes in the price of a commodity is $0.65, the standard deviation of quarterly changes in a futures price on the commodity is $0.81, and the coefficient of correlation between the two changes is 0.8. What is the optimal hedge ratio for a three month contract? What does it mean.

# Chapter 3: Forward and Futures prices

We will cover and discuss about

- Forward prices for an investment asset
- Forward prices for a fixed income paying asset
- Forward prices for a known dividend yield paying asset
- Stock index futures
- Futures on commodities

# Calculating interest rates

| Duration | Interest (%) |
|---|---|
| 7 to14 days | 3.75 |
| 15 to 45 days | 5.00 |
| 46 to 179 days | 5.50 |
| 180 days to less than 1 year | 6.50 |
| 1 year to less than 3 years | 8.25 |
| 3 years to less than 5 years | 8.25 |
| 5 years to upto 10 years | 8.25 |

**Note: This data has the actual figures as per SBI [http://www.statebankofindia.com] and the rates are effective from 22nd Jan, 2007**

# Calculating interest rates

Consider an amount of A invested for n years at an interest rate of r per annum and if the interest rate is compounded once per annum, then the terminal value of wealth is $A(1+r)^n$

# Calculating interest rates

If you have 100 (INR) which you deposit in the bank for 12 months at the interest rate of 8.25% per annum compounded annually, then the terminal value of the wealth is $100*(1+0.0825)^1$, provided you do not take out or deposit any money in this time period of 12 months

# Calculating interest rates

Consider an amount of A invested for n years at an interest rate of r per annum and if the interest rate is compounded m times per annum, then the terminal value of wealth is $A(1+r/m)^{m*n}$

# Calculating interest rates

If you have 100 (INR) which you deposit in the bank for 12 months at the interest rate of 8.25% per annum compounded semi-annually, then the terminal value of the wealth is $100*(1+0.0825/2)^{2*1}$, provided you do not take out or deposit any money in this time period of 12 months

# Calculating interest rates

Consider an amount of A invested for n years at an interest rate of r per annum and if the interest rate is compounded continuously per annum, then the terminal value of wealth is $Ae^{rn}$

# Calculating interest rates

If you have 100 (INR) which you deposit in the bank for 12 months at the interest rate of 8.25% per annum compounded continuously, then the terminal value of the wealth is $100*e^{0.0825*1}$, provided you do not take out or deposit any money in this time period of 12 months.

# Calculating interest rates

Consider an amount of A invested for T number of years at

- an interest rate of $r_1$ per annum (continuous compounding) or
- equivalently at an interest rate of $r_2$ per annum (compounded m times per annum), or
- equivalently at an interest rate of $r_3$ per annum (compounded n times per annum),

Then

$$A*\exp(r_1*T) = A*\{1+(r_2/m)\}^{m*T} = A*\{1+(r_3/n)\}^{n*T}$$

# Calculating interest rates

- A = 100 Euros
- $r_1$ = 4.229% per annum (continuous compounding)
- T = 3 months
- Terminal value = $100*e^{0.04229*(3/12)}$ = 101.0629

Note: This is the actual data as on 23th July, 2007 for Germany and is available on Deutsche BundesBank web site, [http://www.bundesbank.de]

# Calculating interest rates

- A = 100 Euros
- $r_2$ = ? % per annum (compounding being done two times)
- T = 3 months
- Terminal value = $100*e^{0.04229*(3/12)} = 101.0629$
- Hence $r_2 = 4.274\%$

# Calculating interest rates

- A = 100 Euros
- $r_3$ = ? % per annum (compounding being done four times)
- T = 3 months
- Terminal value = $100*e^{0.04229*(3/12)} = 101.0629$
- Hence $r_3 = 4.2515\%$

# Forward and Futures Prices

Important question: How are the forward ad futures prices related to the spot price of the underlying asset?

# Forward and Futures Prices

Forward prices are easier to analyze than Futures price? ***WHY***?
Main reason: There is ***no*** daily settlement for forward contract, but it is present for future contract.
Note: But the prices are very close to each other when the maturities are the same.

# Forward Prices

We will analyze forward prices and that too mainly for investment assets rather than consumption assets.

# Forward Prices

- $F_0$: Forward price today (current forward price)
- $S_0$: Price of asset underlying the forward contract today (current price of asset)
- r: Risk free interest rate with continuous compounding
- T: Time when the forward contract matures and this is in years

# Forward Prices

Forward price for an investment asset providing ***no income (non dividend paying stock)***

$$F_0 = S_0 e^{rT}$$

# Forward Prices

- $S_0$: 327.518 UK Pounds
- r: 5.75% per annum (continuous compounding)
- T: 6 months
- $F_0$: $327.518*e^{0.0575*(6/12)}$

  Note: These are the actual values for gold bullion and risk free interest rate as on 13th July, 2007, obtained from Bank of England [http://www.bankofengland.co.uk]

# Forward Prices

- $F_0 > S_0 e^{rT}$

  Arbitrageurs buy the asset and short forward contracts on the asset

- $F_0 < S_0 e^{rT}$

  Short the asset and buy forward contracts

# Forward Prices

Forward price for an investment asset providing ***a known cash income***

$$F_0 = (S_0 - I)e^{rT}$$

Where

I = Present value of income of the investment asset.

# Forward Prices

Consider a 15 month forward contract on a bond which has a face value of $100 and the risk free interest rate (continuously compounded) is 4.86% per annum for all maturities (i.e., for all time maturities, even though this value is for 6 months). You know that coupons, each of amount $3 for this bond, are paid after 6 and 12 months. The final coupon is paid after 18 months along with principle amount. We are required to find the price of the forward, i.e., $F_0$

Note: The data for the interest is the actual value of risk free interest rate being offered by the US federal government treasury bills as on 19th July, 2007 and the data is available from http://www.federalreserve.gov

# Forward Prices

- $I = 3*e^{-0.0486*(6/12)} + 3*e^{-0.0486*(12/12)}$
  $= 5.79$

- $F_0 = (100 - 5.79)*e^{0.0486*(15/12)}$
  $= 88.66$

Note if there was **no** intermittent coupon, then $F_0 = 100*e^{0.0486*(15/12)} = 106.2633$

# Forward Price

Forward price for an investment asset providing ***a known dividend yield***

$$F_0 = S_0 e^{(r - q)T}$$

where

q = Rate of the continuous dividend yield being provided by the investment

# Forward Price

Dividend yield implies that, when the income is known and when it is expressed as a percentage of the asset price, then this percentage is the dividend yield for that particular asset

# Forward Prices

- A = $100
- r = 4.86% per annum (continuous compounding)
- T = 15 months for the contract
- c (Coupon) = $3 per six months
- Maturity of the actual financial asset, i.e., the bond, is 18 months
- Hence price of bond is

  $3*e^{-0.0486*(6/12)}+3*e^{-0.0486*(12/12)}+(100+3)*e^{-0.0486*(18/12)} = 101.5441$
- Main question is to find q?

# Forward Price

- Buy spot $e^{-qT}$ of asset and reinvest income in the asset.
- Short a forward contract on ONE unit of asset.
- Hence asset grows to $e^{-qT} X e^{qT}$, or become 1 unit of asset in time period T.
- Under the terms of contract the contract is sold for $F_0$ at time T.
- So initial OUTFLOW is $S_0 e^{-qT}$ and final INFLOW is $F_0$.
- As present value of INFLOW and OUTFLOW MUST BE EQUAL, hence $F_0 e^{-rT} = S_0 e^{-qT}$

# Value of Forward Contract

- $f = (F_0 - K)e^{-rT}$ (long forward contract) for an investment or consumption asset with no income. This we can say is some intrinsic value of the difference in the forward contract, between the long and the short position
- $f = S_0 - Ke^{-rT}$ (long forward contract) for an investment that provides no income

# Value of Forward Contract

- $f = S_0 – I – Ke^{-rT}$ (long forward contract) for an investment that provides a known income with present value of I
- $f = S_0e^{-qT} – Ke^{-rT}$ (long forward contract) for an investment that provides a known dividend yield of q

# Foreign Exchange Contract

$$F_0 = S_0 e^{(r - r^*)T}$$

Where

- $F_0$: Forward price measured in domestic currency of one unit of foreign currency
- $S_0$: Current spot price measured in domestic currency of one unit of foreign currency
- r*:  Foreign risk free interest rate

# Value of Forward Foreign Exchange Contract

$$f = S_0 e^{-r^*T} - Ke^{-rT}$$

Now if we have

- $F_0 > S_0 e^{(r-r^*)T}$, then what happens?
- $F_0 < S_0 e^{(r-r^*)T}$, then what happens?

# Stock Index Futures

$$F_0 = S_0 e^{(r-q)T}$$

Examples: DJIA, S&P500, S&P Mid Cap 400, Nikkie 225 Stock Average, NASDAQ 100, CAC 40 Index, FT-SE 100 Index, GSCI, S&P CNX Nifty, CNX Nifty Junior, S&P CNX 500 etc.

# Futures in Commodities

- No Storage cost and provides no income, then

  $F_0 = S_0 e^{rT}$

- Storage cost of value U and provides no income, then

  $F_0 = (S_0 + U)e^{rT}$

- Storage cost proportional to price of commodity, such that u is storage cost per annum as a proportion of the spot price and provides no income, then

  $F_0 = S_0 e^{(r+u)T}$

# Futures in Commodities

For commodities which yield some value due to its consumption we can incorporate the ***convenience yield***. Now if the storage costs per unit are a constant proportion, u, of the spot price, y (the convenience yield) is defined as

$F_0 e^{yT} = S_0 e^{(r+u)*T}$, i.e., $F_0 = S_0 e^{(r+u-y)*T}$

# Summary result for different assets

| Asset | f | $F_0$ |
|---|---|---|
| ▪ Provides no income | $S_0-Ke^{-rT}$ | $S_0e^{rt}$ |
| ▪ Provides known income (I) | $S_0-I-Ke^{-rT}$ | $(S_0-I)e^{rT}$ |
| ▪ Provides known dividend yield (q) | $S_0e^{-qT}-Ke^{-rT}$ | $S_0e^{(r-q)T}$ |

# Example 3.1

Suppose that the risk free interest rate is 10% per annum with continuous compounding and the dividend yield on a stock index is 4% per annum. The index is standing at 400 and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

# Example 3.2

The two month interest rates in Switzerland and the United States with continuous compounding are 3% and 8% per annum, respectively. The spot price of the Swiss franc is $ 0.6500. The futures price for a contract deliverable in two months is $ 0.6600. What arbitrage opportunities does it create?

# Chapter 4: Interest rates and durations

## We will cover and discuss about

- Types of interest rates
- Zero rates
- Bond pricing
- Forward rate calculations
- Interest rate futures
- Treasury rate futures
- Foreign currency futures